EXERGY ANALYSIS

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EXERGY

- definition
- calculation of exergy values (incl. chemical exergy)
- calculation of exergy loss
- exergy efficiencies
- visualization of exergy and exergy loss
EXERGY

PURPOSE OF EXERGY ANALYSIS

• to determine exergy losses (true thermodynamic losses) in processes and systems
• minimisation of losses / optimisation of driving forces

THIS COURSE DISCUSSES

• the calculation of exergy values:
  exergy of heat
  exergy of a flow of matter: thermo-mechanical exergy
  chemical exergy
• calculation of exergy losses in open, steady state systems (\( T_0 \cdot \Delta s \))
• definition of exergy efficiencies (apparatuses, plants)
• visualization of exergy losses and flows in diagrams:
  property diagrams (\( T,s,-, h,s\)-diagrams)
  exergy flow diagrams (Grassmann-diagrams)
  value diagrams
DEFINITION OF EXERGY

GENERAL DEFINITION:
EXERGY = the maximum theoretical work that can be obtained from an amount of energy

A MORE COMPREHENSIVE DEFINITION:
EXERGY = the work that can be obtained from an amount of energy (converted in a well-defined system), under ideal conditions (applying reversible processes), using the environment only as a reservoir of heat and matter
EXERGY ANALYSIS OF OPEN, STEADY STATE, CONSTANT VOLUME SYSTEMS

general layout of real systems (open steady state system with constant volume)

open, steady state, constant volume systems to determine:

- the exergy of heat
- the exergy of (mass) flows

(work is 100% exergy)
EXERGY OF HEAT

To calculate the exergy of heat (at temperature $T$):

- make use of a reversible thermal power cycle (closed cycle)
- heat is discharged only to the environment (at $T_0$)

Maximum theoretical work from a reversible power cycle:

$$dW_{rev} = \left(1 - \frac{T_C}{T_H}\right) \cdot dQ$$

with $T_H = T$ and $T_C = T_0$:

$$dW_{rev} = \left(1 - \frac{T_0}{T}\right) \cdot dQ$$

and

$$dW_{rev} = dEx_Q$$

Thus:

$$dEx_Q = \left(1 - \frac{T_0}{T}\right) \cdot dQ$$
EXERGY OF HEAT

exergy of heat:

\[ dEx_Q = \left(1 - \frac{T_0}{T}\right) \cdot dQ \]

in general, heat will be transferred to a system at varying temperatures:

\[ Ex_Q = \int_{1}^{2} dEx_Q = \int_{1}^{2} \left(1 - \frac{T_0}{T}\right) \cdot dQ \]

or:

\[ Ex_Q = \left(1 - \frac{T_0}{T}\right) \cdot Q \]

\( \bar{T} \) = thermodynamic equivalent temperature of heat transfer to the cycle
EXERGY OF A FLOW OF MATTER

Characteristics of a system used to determine the exergy of a flow of matter:

- system with only reversible processes
- steady state flow in open system with constant volume
- heat is transferred only to and from the environment at $T_0$
- system brings matter into equilibrium with environment

lay-out of a system that brings an amount of substance (or mass flow) into equilibrium with environment:
equilibrium with environment means (in this case):

- system outlet pressure equals $p_0$
- system outlet temperature equals $T_0$

NB
- at system outlet the matter is in thermo-mechanical equilibrium with the environment
- chemical composition of matter remains unchanged
- therefore the exergy determined in this way is called:

**thermo-mechanical exergy**
DETERMINING THE EXERGY OF A FLOW

energy balance of an open steady state system with constant volume:

\[ Q = (H_2 - H_1) + (E_{k,2} - E_{k,1}) + (E_{p,2} - E_{p,1}) + W \]

as:

\[ E_{k,2} \approx E_{k,1} , \ E_{p,2} \approx E_{p,1} \]

\[ H_2 = H_0 , \ H_1 = H \]

Than:

\[ Q = (H_0 - H) + W \] \hspace{1cm} (1)

reversible processes:

\[ Q = \int T \cdot dS \]

heat is supplied at \( T_0 \):

\[ Q = T_0 \cdot (S_0 - S) \] \hspace{1cm} (2)

combining (1) and (2):

\[ W_{\text{rev}} = E_{\text{Ex, matter}} = (H - H_0) - T_0 \cdot (S - S_0) \]

and for a flow of mass:

\[ \Phi_{W_{\text{rev}}} = \Phi_{E_{\text{Ex, flow}}} = \Phi_m \cdot \left[ (h - h_0) - T_0 \cdot (s - s_0) \right] \]
EXERGY LOSS OF AN OPEN, STEADY STATE, CONSTANT VOLUME SYSTEM

exergy balance of an irreversible system in steady state:

\[ \dot{E}x_{\text{loss}} = \dot{E}x_{\text{in}} - \dot{E}x_{\text{out}} \]

for the considered system:

\[ \dot{E}x_{\text{in}} = \dot{E}x_1 + \int_1^2 \left( 1 - \frac{T_0}{T} \right) \cdot dQ \]

\[ \dot{E}x_{\text{out}} = \dot{E}x_2 + \dot{W}_{\text{irrev}} \]

thus:

\[ \dot{E}x_{\text{loss}} = \dot{E}x_1 - \dot{E}x_2 + \int_1^2 \left( 1 - \frac{T_0}{T} \right) \cdot dQ - \dot{W}_{\text{irrev}} \]

with:

\[ \dot{E}x_1 = (H_1 - H_0) - T_0 \cdot (S_1 - S_0) \]

\[ \dot{E}x_2 = (H_2 - H_0) - T_0 \cdot (S_2 - S_0) \]

can be written:

\[ \dot{E}x_1 - \dot{E}x_2 = (H_1 - H_2) - T_0 \cdot (S_1 - S_2) \]
EXERGY LOSS OF AN OPEN, STEADY STATE, CONSTANT VOLUME SYSTEM

combining the results from the previous slide gives:

\[ E_{\text{loss}} = (H_1 - H_2) - T_0 \cdot (S_1 - S_2) + Q - T_0 \cdot \int_{1}^{2} \left( \frac{dQ}{T} \right) - W_{\text{irrev}} \]

from the first law it is known that (energy balance of open steady state system):

\[ Q = (H_2 - H_1) + W_{\text{irrev}} \quad \Rightarrow \quad Q + (H_1 - H_2) - W_{\text{irrev}} = 0 \]

than the exergy balance becomes:

\[ E_{\text{loss}} = T_0 \cdot \left( (S_2 - S_1) - \int_{1}^{2} \left( \frac{dQ}{T} \right) \right) \]

the entropy balance is:

\[ (S_2 - S_1) = \int_{1}^{2} \left( \frac{dQ}{T} \right) + \Delta S_{\text{irrev}} \quad \Rightarrow \quad \Delta S_{\text{irrev}} = (S_2 - S_1) - \int_{1}^{2} \left( \frac{dQ}{T} \right) \]

combining these two equations gives:

\[ E_{\text{loss}} = T_0 \cdot \Delta S_{\text{irrev}} \]
EXERGY EFFICIENCY OF HEAT EXCHANGE

possible definition of heat exchanger efficiency
(*universal efficiency*)

\[
\eta_{Ex, u} = \frac{\sum E_{X_{out}}}{\sum E_{X_{in}}} = \frac{E_{X_2} + E_{X_4}}{E_{X_1} + E_{X_3}}
\]

as \[\sum E_{X_{out}} = \sum E_{X_{in}} - \sum E_{X_{loss}}\]

\[E_{X_2} + E_{X_4} = E_{X_1} + E_{X_3} - E_{X_{loss}}\]

thus \[\eta_{Ex, u} = \frac{E_{X_1} + E_{X_3} - E_{X_{loss}}}{E_{X_1} + E_{X_3}}\]

correct definition of heat exchanger efficiency
(*functional efficiency*)

\[
\eta_{Ex, f} = \frac{\sum E_{X_{product}}}{\sum E_{X_{source}}} = \frac{E_{X_2} - E_{X_1}}{E_{X_3} - E_{X_4}} \quad \Leftrightarrow \quad \eta_{Ex, f} = \frac{E_{X_3} - E_{X_4} - E_{X_{loss}}}{E_{X_3} - E_{X_4}}
\]

note that: \[E_{X_1} + E_{X_3} > E_{X_3} - E_{X_4}\] and therefore: \[\eta_{Ex, u} > \eta_{Ex, f}\]
EXERGY EFFICIENCY HEAT EXCHANGE

universal exergy efficiency (should be avoided):

\[ \eta_{ex,u} = \frac{E_x_2 + E_x_4}{E_x_1 + E_x_3} = \frac{(E_x_1 + E_x_3) - E_{x,loss}}{(E_x_1 + E_x_3)} \]

functional (correct) exergy efficiency:

1. Exergy flows in heat exchanger
2. \[ \eta_{ex,f} = \frac{E_x_2 - E_x_1}{E_x_3 - E_x_4} = \frac{\Delta E_{x,p}}{\Delta E_{x,s}} \]
3. \[ \eta_{ex,f} = \frac{\Delta E_{x,s} - E_{x,loss}}{\Delta E_{x,s}} \]
4. \[ \eta_{ex,f} \leq \eta_{ex,u} \]
EXERGY EFFICIENCIES

CONCLUSION:
only the functional exergy-efficiency is a true thermodynamic efficiency

exergy efficiency:  \[ \eta_{ex} = \frac{\sum E_{x_{product}}}{\sum E_{x_{source}}} = \frac{\sum E_{x_{source}} - E_{x_{loss}}}{\sum E_{x_{source}}} \]

(= functional efficiency)

\( \sum E_{x_{product}} \) and \( \sum E_{x_{source}} \) have to be specified for each type of system

a universal efficiency can be used if a definition of the functional efficiency is not possible (i.e. if no product can be defined)

(universal efficiency):  \[ \eta_{ex, u} = \frac{\sum E_{x_{out}}}{\sum E_{x_{in}}} = \frac{\sum E_{x_{in}} - E_{x_{loss}}}{\sum E_{x_{in}}} \]

\( \sum E_{x_{in}} \) = exergy of energy flows entering the system
\( \sum E_{x_{out}} \) = exergy of energy flows leaving the system

note that:  \[ E_{x_{source}} \leq E_{x_{in}} \quad \Rightarrow \quad \eta_{ex, f} \leq \eta_{ex, u} \]
EXERGY EFFICIENCIES OF EXPANSION (TURBINE) AND COMPRESSION (COMPRESSOR)

exergy efficiency of steam turbines:

$$\eta_{ex} = \frac{E_{X \text{product}}}{E_{X \text{source}}} = \frac{W}{E_{X \text{in}} - \sum E_{X \text{out, steam}}}$$

exergy efficiency of compressors:

$$\eta_{ex} = \frac{E_{X \text{product}}}{E_{X \text{source}}} = \frac{E_{X \text{out}} - E_{X \text{in}}}{W}$$
EXERGY EFFICIENCY COMBUSTION
(adiabatic combustion)

exergy efficiency of adiabatic combustor:

\[ \eta_{ex} = \frac{E_{X_{product}}}{E_{X_{source}}} = \frac{E_{X_{flue gas}}^{tm} - E_{X_{fuel}}^{tm} - E_{X_{air}}^{tm}}{E_{X_{fuel}}^{ch} + E_{X_{air}}^{ch} - E_{X_{flue gas}}^{ch}} \]

with:

- \( E_{X_{flue gas}}^{tm} = E_{X_{flue gas}}^{ch} + E_{X_{flue gas}}^{ch} \) etc.
- \( E_{X_{tm}} = \) thermo-mechanical exergy
- \( E_{X_{ch}} = \) chemical exergy

if fuel and oxidant are supplied at environmental temperature

\[ E_{X_{air}}^{tm} = 0 \quad \text{and} \quad E_{X_{fuel}}^{tm} = 0 \]

and the equation simplifies into:

\[ \eta_{ex} = \frac{E_{X_{flue gas}}^{tm}}{E_{X_{fuel}}^{ch} + E_{X_{air}}^{ch} - E_{X_{flue gas}}^{ch}} \]
EXERGY EFFICIENCIES

CONCLUSION:
the functional efficiency is the only true thermodynamic efficiency
therefore:

exergy efficiency: $\eta_{ex} = \frac{\sum E_{x_{product}}}{\sum E_{x_{source}}} = \frac{\sum E_{x_{source}} - E_{x_{loss}}}{\sum E_{x_{source}}}$

(= functional efficiency)

definition of exergy efficiencies of specific processes see BB wb4302:
course documents/additional information/appendix exergy efficiencies

however in cases where no product can be defined the universal efficiency enables a thermodynamic comparison of alternative systems

universal efficiency: $\eta_{ex, u} = \frac{\sum E_{x_{out}}}{\sum E_{x_{in}}} = \frac{\sum E_{x_{in}} - E_{x_{loss}}}{\sum E_{x_{in}}}$

warning: be very careful with using efficiencies for the assessment of the thermodynamic quality of processes or systems!!!