# EXERGY ANALYSIS

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# EXERGY

- definition
- calculation of exergy values (incl. chemical exergy)
- calculation of exergy loss
- exergy efficiencies
- visualization of exergy and exergy loss

# EXERGY

## PURPOSE OF EXERGY ANALYSIS

- to determine exergy losses (true thermodynamic losses) in processes and systems
- minimisation of losses / optimisation of driving forces

## THIS COURSE DISCUSSES

- the calculation of exergy values:
  - exergy of heat
  - exergy of a flow of matter: thermo-mechanical exergy
    - chemical exergy
- calculation of exergy losses in open, steady state systems ( $T_0 \cdot \Delta s_{irrev}$)
- definition of exergy efficiencies (apparatuses, plants)
- visualization of exergy losses and flows in diagrams:
  - property diagrams (*T,s*-, *h,s*-diagrams)
  - exergy flow diagrams (Grassmann-diagrams)
  - value diagrams

# DEFINITION OF EXERGY

GENERAL DEFINITION:

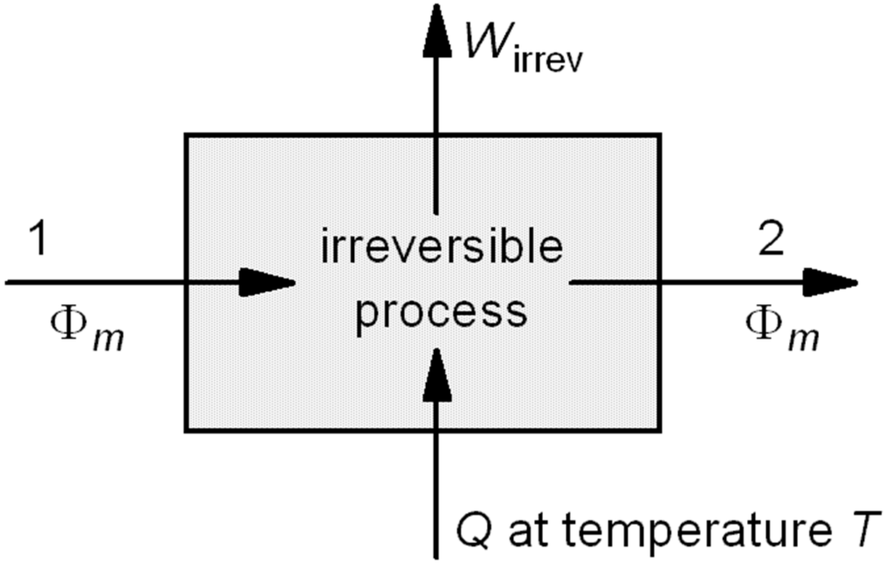
EXERGY = the maximum theoretical work that can be obtained from an amount of energy

A MORE COMPREHENSIVE DEFINITION:

EXERGY = the work that can be obtained from an amount of energy (converted in a well-defined system), under ideal conditions (applying reversible processes), using the environment only as a reservoir of heat and matter

# EXERGY ANALYSIS OF OPEN, STEADY STATE, CONSTANT VOLUME SYSTEMS

general layout of real systems (open steady state system with constant volume)



open, steady state, constant volume systems to determine:

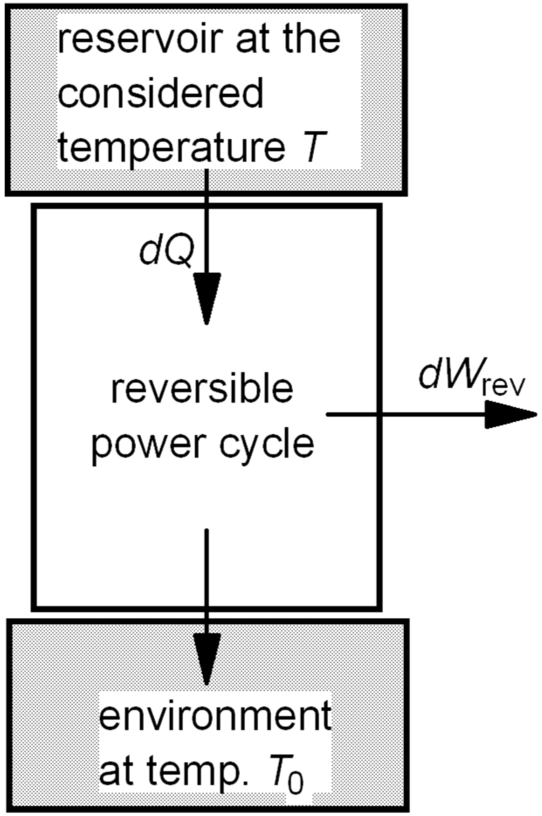
- the exergy of heat
- the exergy of (mass) flows

(work is 100% exergy)

# EXERGY OF HEAT

To calculate the exergy of heat (at temperature $T$):

- make use of a reversible thermal power cycle (closed cycle)
- heat is discharged only to the environment (at $T_0$)



maximum theoretical work from a reversible power cycle:

$$dW_{\text{rev}} = \left(1 - \frac{T_C}{T_H}\right) \cdot dQ$$

with $T_H = T$ and $T_C = T_0$:

$$dW_{\text{rev}} = \left(1 - \frac{T_0}{T}\right) \cdot dQ \quad \text{and} \quad dW_{\text{rev}} = dEx_Q$$

thus: $$dEx_Q = \left(1 - \frac{T_0}{T}\right) \cdot dQ$$

# EXERGY OF HEAT

exergy of heat:

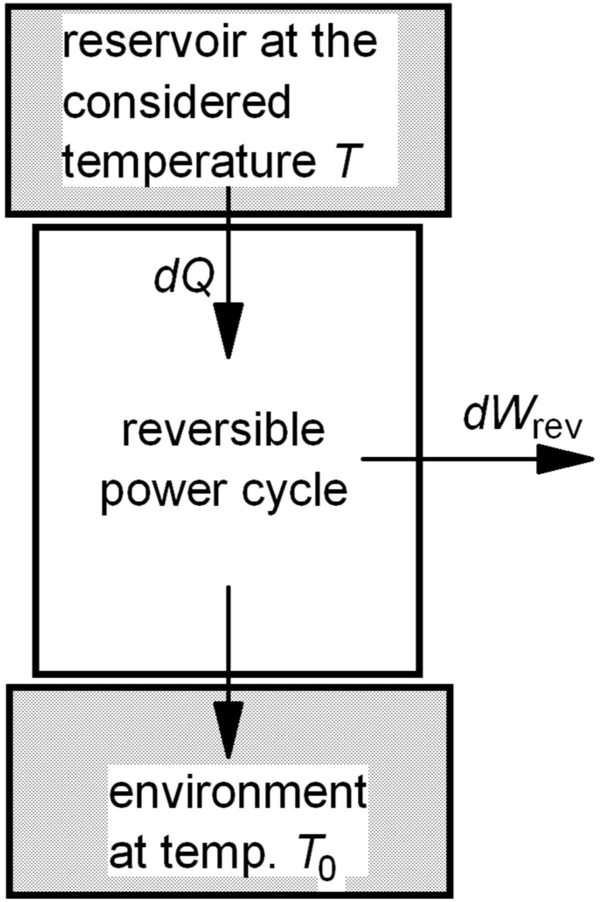
$$dEx_Q = \left(1 - \frac{T_0}{T}\right) \cdot dQ$$

in general, heat will be transferred to a system at varying temperatures:

$$Ex_Q = \int_1^2 dEx_Q = \int_1^2 \left(1 - \frac{T_0}{T}\right) \cdot dQ$$

or: $$Ex_Q = \left(1 - \frac{T_0}{\overline{T}}\right) \cdot Q$$

$\overline{T}$ = thermodynamic equivalent temperature of heat transfer to the cycle



reservoir at the considered temperature $T$

$dQ$

reversible power cycle

$dW_{\text{rev}}$

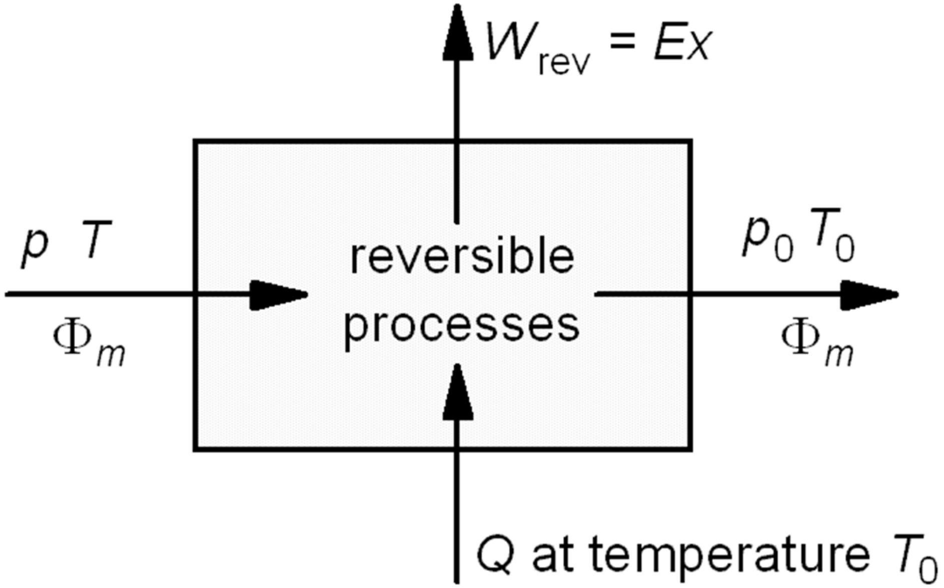
environment at temp. $T_0$

# EXERGY OF A FLOW OF MATTER

Characteristics of a system used to determine the exergy of a flow of matter:

- system with only reversible processes
- steady state flow in open system with constant volume
- heat is transferred only to and from the environment at $T_0$
- system brings matter into equilibrium with environment

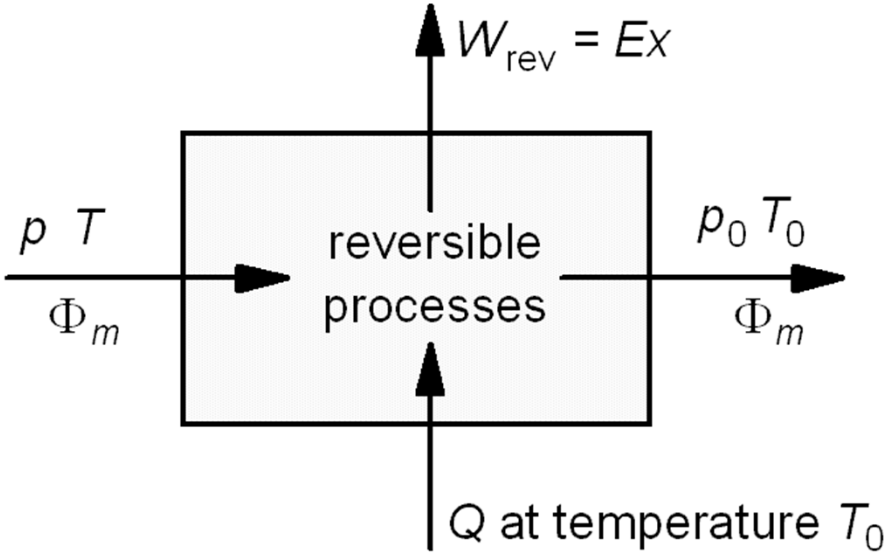
lay-out of a system that brings an amount of substance (or mass flow) into equilibrium with environment:



# EXERGY OF A FLOW OF MATTER

equilibrium with environment means (in this case):

- system outlet pressure equals $p_0$
- system outlet temperature equals $T_0$



NB

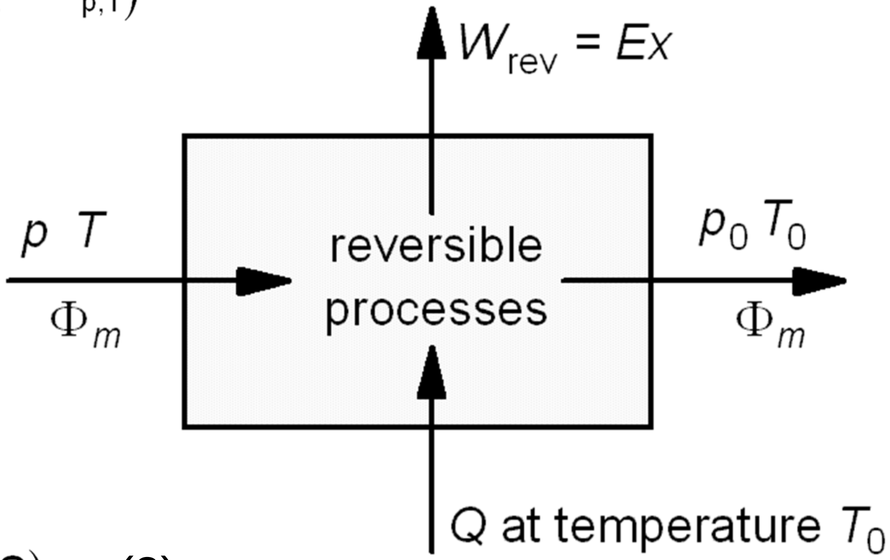
- at system outlet the matter is in thermo-mechanical equilibrium with the environment
- chemical composition of matter remains unchanged
- therefore the exergy determined in this way is called:

***thermo-mechanical exergy***

# DETERMINING THE EXERGY OF A FLOW

energy balance of an open steady state system with constant volume:

$$Q = \left(H_2 - H_1\right) + \left(E_{\mathrm{k},2} - E_{\mathrm{k},1}\right) + \left(E_{\mathrm{p},2} - E_{\mathrm{p},1}\right) + W$$



as: $E_{\mathrm{k},2} \approx E_{\mathrm{k},1} \ , \ E_{\mathrm{p},2} \approx E_{\mathrm{p},1}$

$H_2 = H_0 \ , \ H_1 = H$

than: $Q = \left(H_0 - H\right) + W$ (1)

reversible processes: $Q = \int T \cdot dS$

heat is supplied at $T_0$: $Q = T_0 \cdot \left(S_0 - S\right)$ (2)

combining (1) and (2):

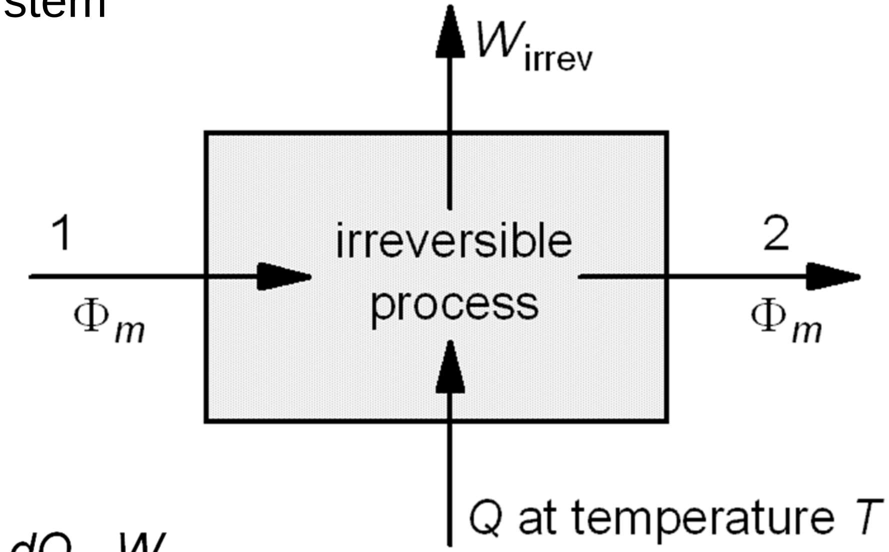
$$W_{\mathrm{rev}} = Ex_{\mathrm{matter}} = \left(H - H_0\right) - T_0 \cdot \left(S - S_0\right)$$

and for a flow of mass: $\Phi_{W_{\mathrm{rev}}} = \Phi_{Ex_{\mathrm{flow}}} = \Phi_m \cdot \left[\left(h - h_0\right) - T_0 \cdot \left(s - s_0\right)\right]$

# EXERGY LOSS OF AN OPEN, STEADY STATE, CONSTANT VOLUME SYSTEM

exergy balance of an irreversible system in steady state:

$$Ex_{\text{loss}} = Ex_{\text{in}} - Ex_{\text{out}}$$



for the considered system:

$$Ex_{\text{in}} = Ex_1 + \int_1^2 \left(1 - \frac{T_0}{T}\right) \cdot dQ$$

$$Ex_{\text{out}} = Ex_2 + W_{\text{irrev}}$$

thus: $$Ex_{\text{loss}} = Ex_1 - Ex_2 + \int_1^2 \left(1 - \frac{T_0}{T}\right) \cdot dQ - W_{\text{irrev}}$$

with: $$Ex_1 = \left(H_1 - H_0\right) - T_0 \cdot \left(S_1 - S_0\right)$$

$$Ex_2 = \left(H_2 - H_0\right) - T_0 \cdot \left(S_2 - S_0\right)$$

can be written: $$Ex_1 - Ex_2 = \left(H_1 - H_2\right) - T_0 \cdot \left(S_1 - S_2\right)$$

# EXERGY LOSS OF AN OPEN, STEADY STATE, CONSTANT VOLUME SYSTEM

combining the results from the previous slide gives:

$$Ex_{\text{loss}} = \left(H_1 - H_2\right) - T_0 \cdot \left(S_1 - S_2\right) + Q - T_0 \cdot \int_1^2 \left(\frac{dQ}{T}\right) - W_{\text{irrev}}$$

from the first law it is known that (energy balance of open steady state system):

$$Q = \left(H_2 - H_1\right) + W_{\text{irrev}} \qquad \Rightarrow \qquad Q + \left(H_1 - H_2\right) - W_{\text{irrev}} = 0$$

than the exergy balance becomes: $Ex_{\text{loss}} = T_0 \cdot \left(\left(S_2 - S_1\right) - \int_1^2 \left(\frac{dQ}{T}\right)\right)$

the entropy balance is:

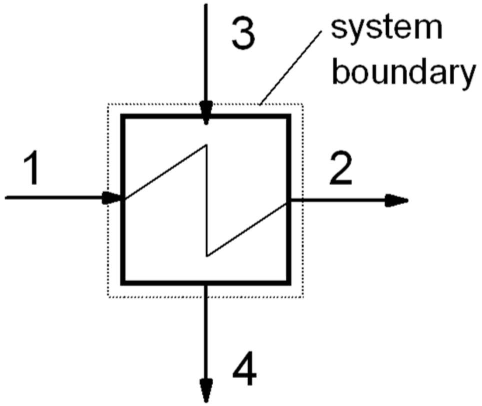
$$\left(S_2 - S_1\right) = \int_1^2 \left(\frac{dQ}{T}\right) + \Delta S_{\text{irrev}} \qquad \Rightarrow \qquad \Delta S_{\text{irrev}} = \left(S_2 - S_1\right) - \int_1^2 \left(\frac{dQ}{T}\right)$$

combining these two equations gives: $\boxed{Ex_{\text{loss}} = T_0 \cdot \Delta S_{\text{irrev}}}$

# EXERGY EFFICIENCY OF HEAT EXCHANGE

possible definition of heat exchanger efficiency
(*universal efficiency*)

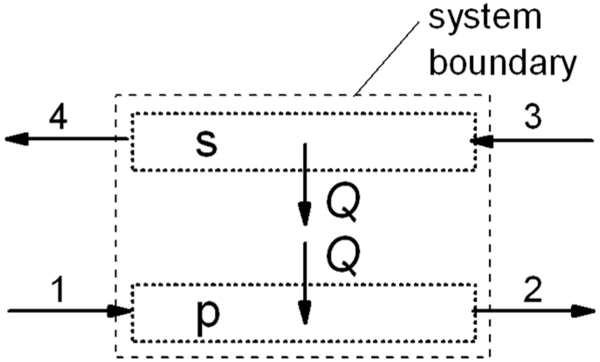
$$\eta_{Ex,\,\mathrm{u}} = \frac{\sum Ex_{\mathrm{out}}}{\sum Ex_{\mathrm{in}}} = \frac{Ex_2 + Ex_4}{Ex_1 + Ex_3}$$



as $\sum Ex_{\mathrm{out}} = \sum Ex_{\mathrm{in}} - \sum Ex_{\mathrm{loss}}$

$$Ex_2 + Ex_4 = Ex_1 + Ex_3 - Ex_{\mathrm{loss}}$$

thus $\eta_{Ex,\,\mathrm{u}} = \dfrac{Ex_1 + Ex_3 - Ex_{\mathrm{loss}}}{Ex_1 + Ex_3}$



correct definition of heat exchanger efficiency
(*functional efficiency*)

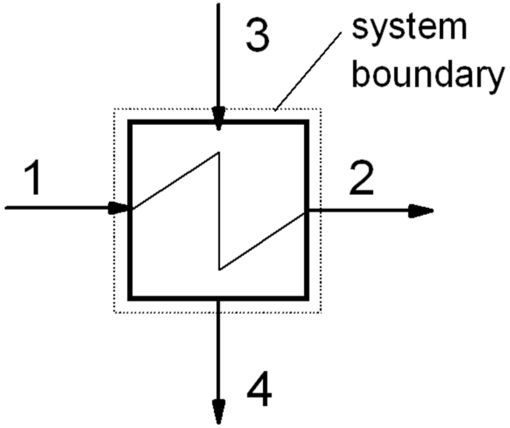
$$\eta_{Ex,\,\mathrm{f}} = \frac{\sum Ex_{\mathrm{product}}}{\sum Ex_{\mathrm{source}}} = \frac{Ex_2 - Ex_1}{Ex_3 - Ex_4} \quad \Rightarrow \quad \eta_{Ex,\,\mathrm{f}} = \frac{Ex_3 - Ex_4 - Ex_{\mathrm{loss}}}{Ex_3 - Ex_4}$$

note that: $Ex_1 + Ex_3 > Ex_3 - Ex_4$ and therefore: $\eta_{Ex,\,\mathrm{u}} > \eta_{Ex,\,\mathrm{f}}$

# EXERGY EFFICIENCY HEAT EXCHANGE

universal exergy efficiency (should be avoided):

$$\eta_{ex,\,\mathrm{u}} = \frac{Ex_2 + Ex_4}{Ex_1 + Ex_3} = \frac{\left(Ex_1 + Ex_3\right) - Ex_{\mathrm{loss}}}{\left(Ex_1 + Ex_3\right)}$$



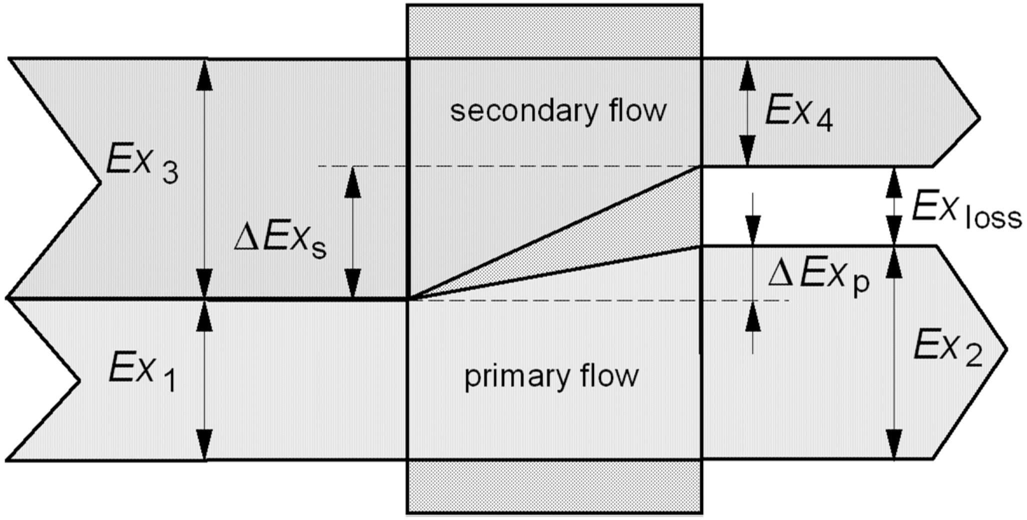
functional (correct) exergy efficiency:

$$\eta_{ex,\,\mathrm{f}} = \frac{Ex_2 - Ex_1}{Ex_3 - Ex_4} = \frac{\Delta Ex_{\mathrm{p}}}{\Delta Ex_{\mathrm{s}}}$$

$$\eta_{ex,\,\mathrm{f}} = \frac{\Delta Ex_{\mathrm{s}} - Ex_{\mathrm{loss}}}{\Delta Ex_{\mathrm{s}}}$$

$$\eta_{ex,\,\mathrm{f}} \le \eta_{ex,\,\mathrm{u}}$$

exergy flows in heat exchanger



# EXERGY EFFICIENCIES

CONCLUSION:
only the functional exergy-efficiency is a true thermodynamic efficiency

exergy efficiency:
(= functional efficiency)

$$\eta_{ex} = \frac{\sum Ex_{\text{product}}}{\sum Ex_{\text{source}}} = \frac{\sum Ex_{\text{source}} - Ex_{\text{loss}}}{\sum Ex_{\text{source}}}$$

$\Sigma\, Ex_{\text{product}}$ and $\Sigma\, Ex_{\text{source}}$ have to be specified for each type of system

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a universal efficiency can be used if a definition of the functional efficiency is not possible (i.e. if no product can be defined)

(universal efficiency):

$$\eta_{ex,\,\text{u}} = \frac{\sum Ex_{\text{out}}}{\sum Ex_{\text{in}}} = \frac{\sum Ex_{\text{in}} - Ex_{\text{loss}}}{\sum Ex_{\text{in}}}$$

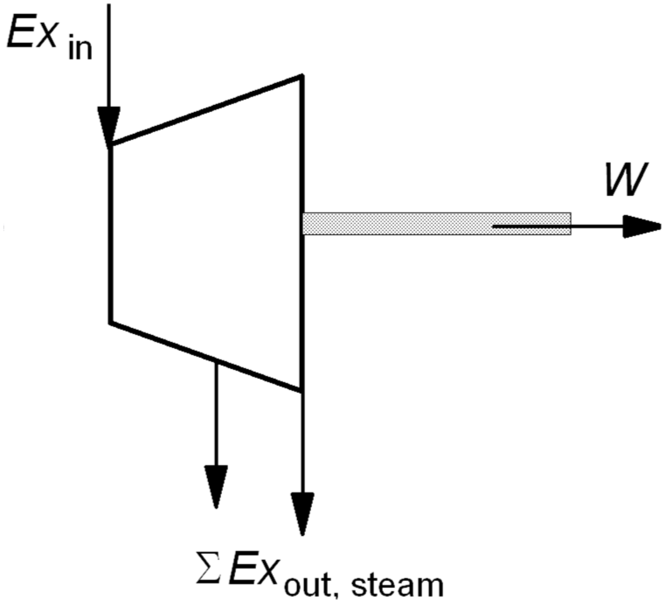
$\Sigma\, Ex_{\text{in}}$ = exergy of energy flows entering the system
$\Sigma\, Ex_{\text{out}}$ = exergy of energy flows leaving the system

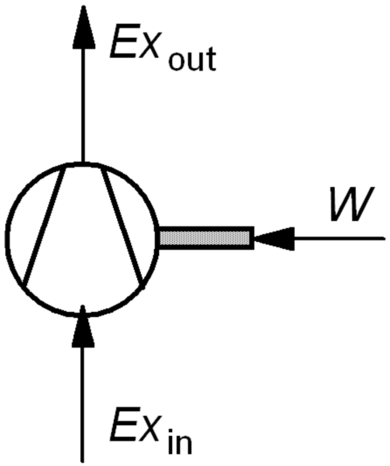
note that: $Ex_{\text{source}} \leq Ex_{\text{in}} \quad \Rightarrow \quad \eta_{ex,\,\text{f}} \leq \eta_{ex,\,\text{u}}$

# EXERGY EFFICIENCIES OF EXPANSION (TURBINE) AND COMPRESSION (COMPRESSOR)

exergy efficiency of steam turbines:

$$\eta_{ex} = \frac{Ex_{\text{product}}}{Ex_{\text{source}}} = \frac{W}{Ex_{\text{in}} - \sum Ex_{\text{out, steam}}}$$



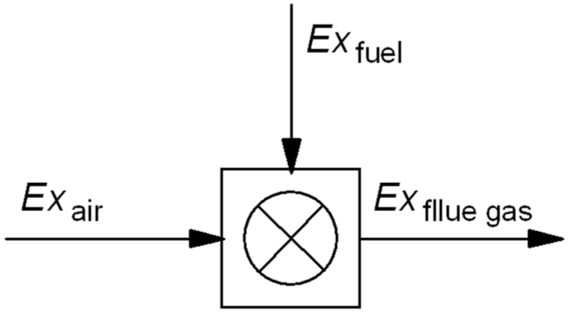


exergy efficiency of compressors:

$$\eta_{ex} = \frac{Ex_{\text{product}}}{Ex_{\text{source}}} = \frac{Ex_{\text{out}} - Ex_{\text{in}}}{W}$$

# EXERGY EFFICIENCY COMBUSTION

## (adiabatic combustion)



exergy efficiency of adiabatic combustor:

$$\eta_{ex} = \frac{Ex_{\text{product}}}{Ex_{\text{source}}} = \frac{Ex^{tm}_{\text{flue gas}} - Ex^{tm}_{\text{fuel}} - Ex^{tm}_{\text{air}}}{Ex^{ch}_{\text{fuel}} + Ex^{ch}_{\text{air}} - Ex^{ch}_{\text{flue gas}}}$$

with: $Ex_{\text{flue gas}} = Ex^{tm}_{\text{flue gas}} + Ex^{ch}_{\text{flue gas}}$ etc.

$Ex^{tm}$ = thermo-mechanical exergy
$Ex^{ch}$ = chemical exergy

if fuel and oxidant are supplied at environmental temperature

$$Ex^{tm}_{\text{air}} = 0 \quad \text{and} \quad Ex^{tm}_{\text{fuel}} = 0$$

and the equation simplifies into:

$$\eta_{ex} = \frac{Ex^{tm}_{\text{flue gas}}}{Ex^{ch}_{\text{fuel}} + Ex^{ch}_{\text{air}} - Ex^{ch}_{\text{flue gas}}}$$

# EXERGY EFFICIENCIES

CONCLUSION:
the functional efficiency is the only true thermodynamic efficiency

therefore:

exergy efficiency:
(= functional efficiency)

$$\eta_{ex} = \frac{\sum Ex_{\text{product}}}{\sum Ex_{\text{source}}} = \frac{\sum Ex_{\text{source}} - Ex_{\text{loss}}}{\sum Ex_{\text{source}}}$$

definition of exergy efficiencies of specific processes see BB wb4302:
course documents/additional information/appendix exergy efficiencies

however in cases where no product can be defined the universal efficiency enables a thermodynamic comparison of alternative systems

universal efficiency:

$$\eta_{ex,\,\text{u}} = \frac{\sum Ex_{\text{out}}}{\sum Ex_{\text{in}}} = \frac{\sum Ex_{\text{in}} - Ex_{\text{loss}}}{\sum Ex_{\text{in}}}$$

warning: be very careful with using efficiencies for the assessment of the thermodynamic quality of processes or systems!!!